PHYS 10352

PART I: CLASSICAL THERMODYNAMICS

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Week 2: The 1st Law of Thermodynamics, Contd.

Recap

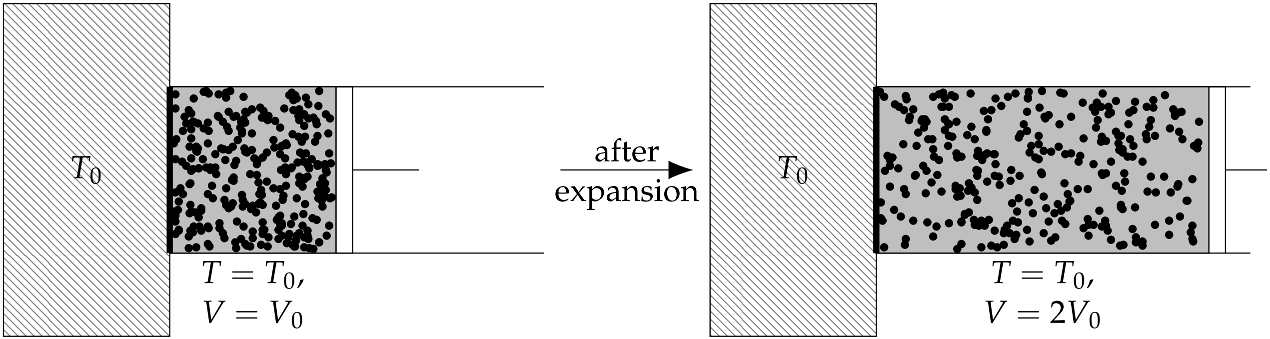
- Ideal gases: point-like particles; elastic collisions; no interactions;
- Real gases behave like ideal gases at high temperatures and low pressures;
- The ideal gas law:

$$PV = nRT$$

- Reversible processes: quasistatic; no external friction; no permanent change to the system;
- The 1st Law of Thermodynamics:

$$dE = đQ + đW$$

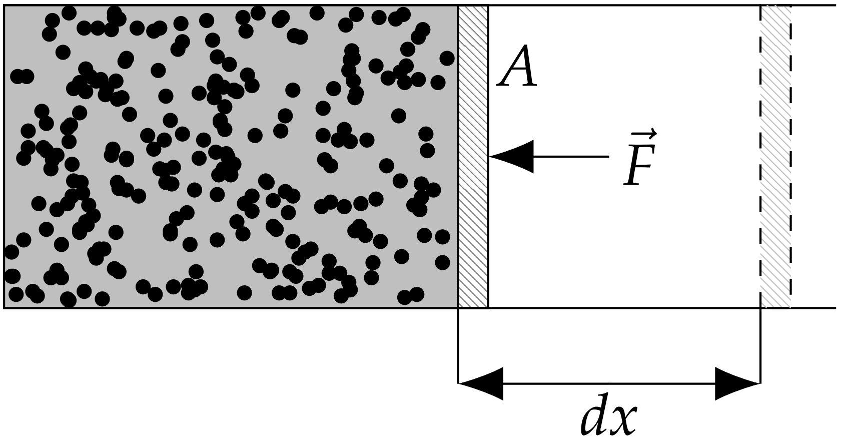
The 1st law of thermodynamics
Calculating the work done



What is the *work done* in this expansion?

The 1st law of thermodynamics

Calculating the work done



$$đW = \vec{F} \cdot \vec{dx} = \underbrace{\frac{|\vec{F}|}{A}}_{P} \underbrace{A\,dx}_{-dV} = -PdV.$$

The tension, Γ, in an elastic string stretched to length L at temperature T is given by

$$\Gamma = KT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right),$$

where $K = 0.1\,\mathrm{N\,K^{-1}}$ and $L_0 = 0.1\,\mathrm{m}$ is the unstretched length.

Calculate the work done on the string when it is stretched *reversibly* and *isothermally* from a length of $0.1\,\mathrm{m}$ to $0.2\,\mathrm{m}$ at $T = 273\,\mathrm{K}$.

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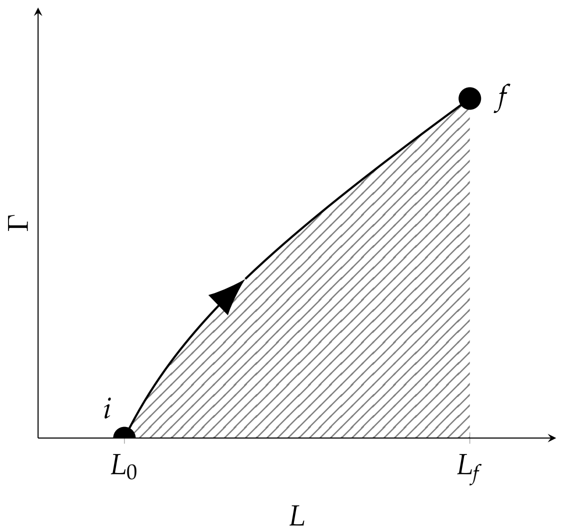
The infinitesimal[1] work done in stretching the string is given by: $đW = \Gamma \mathrm{d}L$.
Since the process is *isothermal*, we can treat T as a constant and calculate the total work done as

$$W = \int_{L_0}^{2L_0} đW = \int_{L_0}^{2L_0} KT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right)\mathrm{d}L = KT\left[\frac{L^2}{2L_0} + \frac{L_0^2}{L}\right]_{L_0}^{2L_0} = 2.73\,\mathrm{J}.$$

[1] very *very* small.

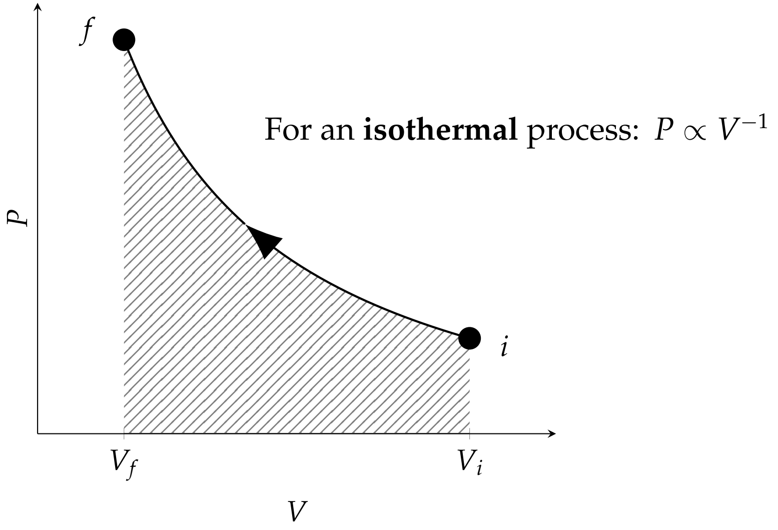
ELASTIC STRING

$$\Gamma = KT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right)$$



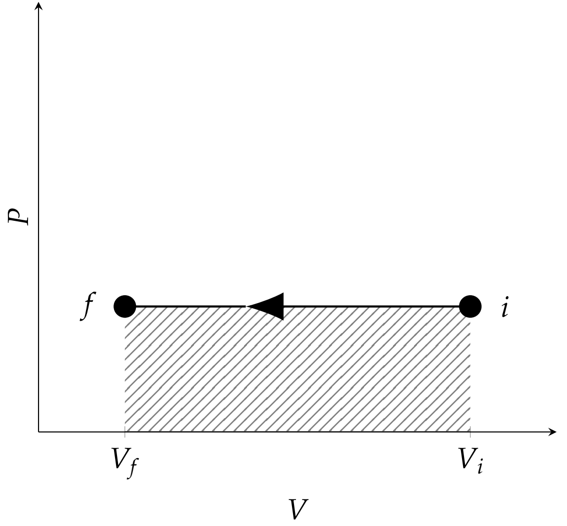
The 1st law of thermodynamics

Indicator diagrams

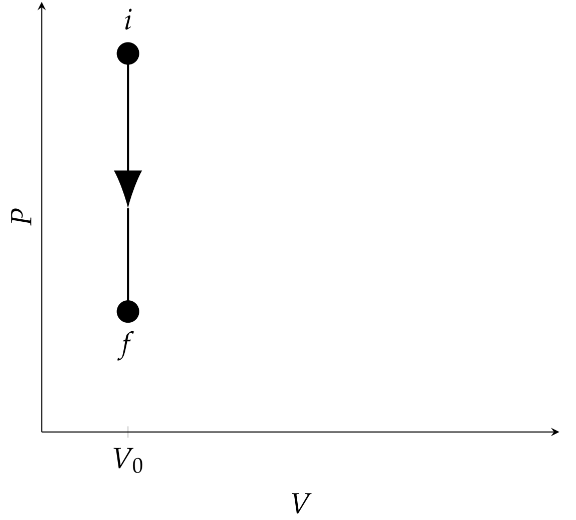


$$W = \int_i^f đW = -\int_{V_i}^{V_f} PdV = \int_{V_f}^{V_i} PdV.$$

An **isobaric** process happens at constant pressure



An **isochoric** process happens at constant volume



Note: Isochoric processes are also referred to as *isometric* processes and *isovolumetric* processes.

The *heat capacity*, C, relates the amount of heat required to enact a given temperature change:

$$đQ = C\,\mathrm{d}T$$

More frequently we use the *specific* heat capacity, c:

$$đQ = mc\,\mathrm{d}T$$

C:	heat capacity	$\mathrm{J\,K^{-1}}$
c:	specific heat capacity	$\mathrm{J\,K^{-1}\,kg^{-1}}$

THE 1ST LAW OF THERMODYNAMICS

SPECIFIC HEAT CAPACITIES

The amount of heat required to produce a given temperature change will differ depending on which other thermodynamic variables are held constant. We can see this by rearranging the equation for the 1st law of thermodynamics:

$$đQ = \mathrm{d}E - đW = \mathrm{d}E + P\mathrm{d}V.$$

• In the case where we have an **isochoric** process (i.e. constant volume), $\mathrm{d}V = 0$, so all of the heat is used to produce the temperature change, i.e.

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V.$$

• In the case where we have an **isobaric** process (i.e. constant pressure), $\mathrm{d}V \neq 0$, so some of the heat is used to do work, i.e.

$$\begin{aligned} C_P &= \left(\frac{\partial E}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P \\ &= \left(\frac{\partial E}{\partial T}\right)_P + nR \quad \underline{\text{for an ideal gas}}, \text{ where } E \equiv E(T). \end{aligned}$$

Derivations for these quantities are given in the notes.

For an ideal gas:

$$\begin{aligned} C_P &= \left(\frac{\partial E}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P \\ &= \left(\frac{\partial E}{\partial T}\right)_P + nR \quad \underline{\text{for an ideal gas}},\ \text{where } E \equiv E(T). \end{aligned}$$

Because $E \equiv E(T)$ for an ideal gas, we can also write:

$$\left(\frac{\partial E}{\partial T}\right)_P = \left(\frac{\partial E}{\partial T}\right)_V \equiv C_V(T) \qquad \Rightarrow \quad \boxed{\mathrm{d}E = C_V \mathrm{d}T}$$

Therefore we find:

$$C_P(T) = C_V(T) + nR,$$

which means that:

$$C_P(T) > C_V(T).$$

In an **adiabatic** process there is no heat transfer $\therefore Q = 0$.

This is equivalent to saying that the system is **thermally insulated** or **isolated**.

Starting from the 1st law of thermodynamics it is possible to show that for an adiabatic process:

$$PV^{\gamma} = k_1$$

$$TV^{\gamma-1} = k_2$$

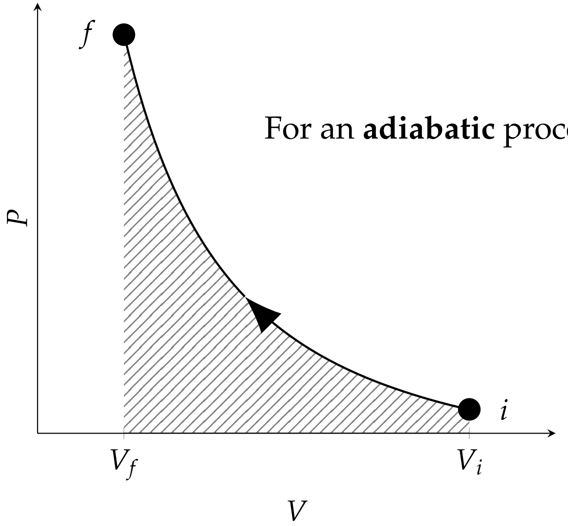
where $k_{1,2}$ are constants and γ is the *adiabatic index*:

$$\gamma = \frac{C_P}{C_V}.$$

A derivation for these properties is given in the course notes.

The 1st law of thermodynamics

Adiabatic Processes



For an **adiabatic** process: $P \propto V^{-\gamma}$

$$W = \int_i^f đW = -\int_{V_i}^{V_f} kV^{-\gamma} dV = k \int_{V_f}^{V_i} V^{-\gamma} dV.$$

The 1st law of thermodynamics

Worked Example - Lightning Strike

⚡ The electrical work done by lightning striking a piece of conductor is given by

$$W = \rho \frac{\ell}{A_S} \int I^2 \mathrm{d}t,$$

where $\int I^2 \mathrm{d}t$ is known as the *action integral* of the lightning strike and is measured in units of $\mathrm{A}^2{\cdot}\mathrm{s}$, ℓ is the depth of the conductor, A_S is the cross-sectional area, and ρ is the resistivity of the conductor.

The landing gear of an aircraft has a width of 100 mm and is made of **[insert conductor here]**. It is struck by lightning with an action integral of $2 \times 10^7\ \mathrm{A}^2\,\mathrm{s}$. Assuming that the conductor can be approximated as an ideal gas, what will be the change in temperature of the landing gear if it is struck by lightning?

	Aluminium	Copper	Titanium	Steel	Magnesium	Silver
Resistivity [$10^{-6}\ \Omega{\cdot}\mathrm{cm}$]	2.8	1.7	42.0	72.0	4.5	1.6
Density [$10^{-3}\mathrm{g}{\cdot}\mathrm{cm}^{-3}$]	4.3	3.9	3.5	1.0	16.5	4.1
Specific heat capacity [$\mathrm{J}{\cdot}\mathrm{g}^{-1}{\cdot}\mathrm{K}^{-1}$]	0.90	0.39	0.52	0.50	1.03	0.23
Melting point [°C]	660	1084	1670	1150	650	962

https://ntrs.nasa.gov/api/citations/19780003081/downloads/19780003081.pdf (p. 189-190)

The duration of a lightning strike is only a few micro-seconds, consequently there is no time for any heat to be radiated away and complete adiabatic conditions can be assumed (i.e. $Q = 0$). In this case, all of the work done goes into changing the internal energy of the conductor, i.e. $\Delta E = W$.

Because we are treating the conductor as an ideal gas, we also know that $\Delta E = C\Delta T$, and therefore:

$$\Delta T = \frac{W}{C} = \frac{\rho \int I^2 \mathrm{d}t}{c \cdot \delta \cdot A_\mathrm{S}^2},$$

where δ here is the density. For example, for steel:

$$\Delta T = \frac{\rho \int I^2 \mathrm{d}t}{c \cdot \delta \cdot A_\mathrm{S}^2} = \frac{72 \times 10^{-6} \cdot 2 \times 10^7}{0.120 \cdot 1 \times 10^{-3} \cdot 10^2} \simeq 287\,\mathrm{K}.$$

	Aluminium	Copper	Titanium	Steel	Magnesium	Silver
ΔT [K]	1.45	2.26	46.26	286.81	0.53	3.33

- What material would you prefer the landing gear to be made of?
- What would happen to a component with a smaller cross-section? i.e. pipes, bond straps etc.

Note: Watch your units! Be aware of SI conversions: $1\,\Omega = 1\,\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-3} \cdot \mathrm{A}^{-2}$; $1\,\mathrm{J} = 1\,\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-2}$

Summary

- The work done on a P–V system is given by $đW = -P\mathrm{d}V$.
- An **isothermal** process happens at *constant temperature* $\Rightarrow \Delta E = 0$ for an ideal gas.
- An **isobaric** process happens at *constant pressure*.
- An **isochoric** process happens at *constant volume* $\Rightarrow đW = 0$.
- An **adiabatic** process happens with *no heat flow* $\Rightarrow đQ = 0$.
- The **heat capacity** relates heat flow to temperature change: $đQ = C\,\mathrm{d}T$.
- Heat capacity is <u>different</u> for constant volume and constant pressure processes, and $C_P(T) > C_V(T)$.
- For an ideal gas: $C_P(T) = C_V(T) + nR$.
- Indicator diagrams allow us to visualise the changes resulting from different types of process.

Final Slide

About 20 km South of Niigata City, on the shore of the Sea of Japan, there is a place called Maki.

In this isolated spot, about 400 m from the shoreline, there is a 150 m tall tower.

What was recorded here on 30 October 1986?